BABHTA 1   ROUND 1

1)   Calculate, in its simplest form, the value of i^{2014}, where $i = \sqrt{-1}$.

2)   Find the numerical value of the derivative of Sin(3x) when x = $\frac{\pi}{3}$.

BABHTA 2   ROUND 2

1)   What is the numerical value $\text{Cos}(15°)\text{Sin}(15°)$?
      Answer in the form $\frac{a}{b}$, where a and b $\in N$

2)   Find the range of values of x for which $3 > 5x + 2x^2, x \in R$

BABHTA 3   ROUND 3

1)   Find the length of the tangents from $(-9, 3)$ to the circle

$$x^2 + y^2 - 4x - 2y - 20 = 0$$

2)   If $3^x = 12$, calculate the value of 9^{x-1}.

BABHTA 4   ROUND 4

1)   Find the fifth term in the arithmetic sequence whose first term is 1, whose common difference is non-zero and whose second, tenth and thirty fourth term are the first three terms of a geometric sequence.

Answer in the form $\dfrac{a}{b}$, where a and $b \in \mathbb{N}$ and in its lowest terms.

2)   Three vertices of a parallelogram are $(1, 1)$, $(3, 5)$ and $(-1, 4)$.
Find all possible ordered pairs that could be the coordinates of the fourth vertex.

BABHTA 5   ROUND 5

1)   Solve for x : $2.3^{2x+1} - 5.3^x - 6 = 0$.

     Answer correct to 2 decimal places.

2)   Find the value of x If $\log_2(\log_2(\log_2(x))) = 2$

     Answer in the form a^b, where a and b $\in$ N, and b is the smallest integer greater than 1

BABHTA 6   ROUND 6

1)   Two identical bottles are filled with weak milk solutions. In one bottle the ratio of the volume of milk to the volume of water is 1 : 25. In the other bottle the ratio of the volume of milk to the volume of water is 1 : 77.
If the entire contents of the two bottles are mixed together, the ratio of the volume of milk to the volume of water in the mixture is 1 : N.
Find the value of N.

2)   Each of the first six prime numbers is written on a separate card.
The cards are shuffled and two cards are selected without replacement. What is the probability that the sum of the numbers selected is prime and unique?
Answer in the form $\dfrac{a}{b}$, where a and b $\in$ N and in its lowest terms.

BABHTA 7   ROUND 7

1)   If $x_1, x_2, x_3, \ldots, x_n$ are real numbers and $x_n = \dfrac{n}{x_{n-1}}$ for $n \geq 1$,

where $n \in \mathbb{N}$, calculate the product $x_1 x_2 x_3 \ldots x_8$.

2)   Find the numerical value of $25^{\frac{1}{2} - \log_5(\sqrt{2})}$

Answer in the form $\dfrac{a}{b}$, where a and $b \in \mathbb{N}$ and in its lowest terms.

3)   Calculate the value of the positive integer n where

$$2014^{2013} < 2013^n < 2014^{2014}$$

4)   Three cubes of volume 1, 8 and 27 are glued together at their face.
Calculate the smallest possible surface area of the resulting configuration.

BABHTA 8   ROUND 8

1) A tennis player computes her "win ratio" by dividing the number of matches she has won by the total number of matches played.
   At the start of a weekend, her win ratio was exactly 0·5.
   During the weekend she plays four matches, winning three and losing one. At the end of the weekend her "win ratio" is greater than 0·503.
   What is the largest number of matches that she could have won before the weekend began?

2)   The numbers 1447, 1005 and 1231 have something in common: each is a four digit number beginning with 1 that has exactly two identical digits.
   How many such numbers are there?

3)   The architecture of a sculpture in a certain city is based on frames as shown in which a large triangle is subdivided into 5 identical triangles, each similar to the large triangle.
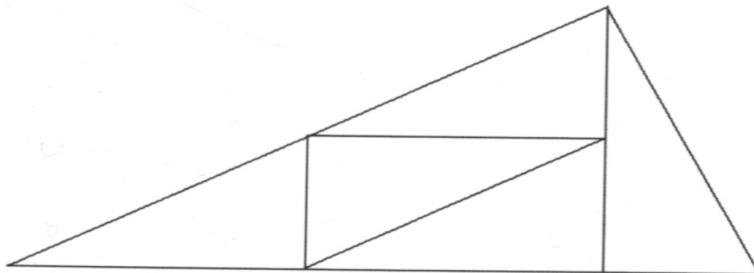
   If the shortest side of one of the smallest triangles is 1 metre,   how many metres of framing are required to construct the whole shape?
   Answer in the form $a + b\sqrt{c}$, where a, b and $c \in$ N

4)   The product of a two-digit number and the same number with its digits reversed is 3154. Find the sum of the two numbers.

Tiebreak

1) If $3^m = 7$ and $7^n = 81$ what is the value of the product mn?

2) What is the shortest distance between the curves $y = 4x^2 + 2$ and $y = -3x^2 - 4$?

3) What is the sum of all three-digit integers whose units digit is 7?

4) An unbiased six sided die is tossed three times.
   Given that the sum of the first two tosses equals the third, what is the probability that at least one "2" is tossed?

5) Suppose that the temperature during June is normally distributed with a mean of 20^0C and a standard deviation of $3 \cdot 33$ degrees. Find the probability that the temperature is between $21 \cdot 11^0$ and $26 \cdot 66^0$.
   Answer correct to 2 decimal places.

6) If $0^0 < A < 90^0$ and $\text{Sin}(A) = 0.1$, find the value of $\log_{10}(\text{Tan}(A)) + \log_{10}(\text{Cos}(A))$

7) Find the numerical value of $x^3 + y^3$, where

$$x + y = 1$$
$$\text{and} \qquad x^2 + y^2 = 2$$

   Answer in the form $\dfrac{a}{b}$, where a and b $\in$ N

8) What is the sum of the digits of the product $2^{2012} \times 5^{2014}$?

9) The addition below is incorrect. What is the largest digit that can be changed in order to make the left hand side equal to the right hand side.

$$641 + 852 + 973 = 2456$$

10) Find the value of k such that the product $(3 + 2i)(3 + ki)$ is a real number.

11) The line $y = \dfrac{1}{2}$ intersects the graph of $y = Sin(x)$ at A and B.

Find the length of the line segment [AB] in the domain $[0, \pi]$

Answer in terms of π, in simplest form.

12) Each of the numbers 2, 5, 11 and 13 are assigned in some order to p, q, r and s.
What is the largest possible value of the $pq + pr + ps$?

13) The base of a triangle is tripled and the altitude is halved. Calculate the ratio of the area of the new triangle to the area of the original triangle.

Answer in the form $a : b$ where a and $b \in N$.

14) Simplify the following expression $1 - \dfrac{1}{1 - \dfrac{1}{1 - \dfrac{1}{2014}}}$.

15) Take any two -digit number **ab**. If we reverse the digits we get the two-digit number **ba**.
If we now make a four-digit number of the form **abba,** what is the smallest positive two-digit integer, that always divides **abba** for all a and b $\in N$?

ANSWER KEY

ROUND 1	Q 1 −1	Q2 − 3		
ROUND 2	Q1 ¼	Q2 $-3 < x < \frac{1}{2}$		
ROUND 3	Q1 10	Q2 16		
ROUND 4	Q1 $\frac{7}{3}$	Q2 (−3, 0), (1, 8), (5, 2)		
ROUND 5	Q1 0·37	Q2 256^2		
ROUND 6	Q1 N=38	Q2 $\frac{1}{5}$		
ROUND 7	Q1 384	Q2 $\frac{5}{2}$	Q3 2014	Q4 72
ROUND 8	Q1 164	Q2 432	Q3 $10+4\sqrt{5}$	Q4 121

TIEBREAK

Q1   4
Q2   6
Q3   49680
Q4   8/15
Q5   0·35
Q6   −1
Q7   5/2
Q8   7
Q9   7
Q10   −2
Q11   2π/3
Q12   234
Q13   3 : 2
Q14   2014
Q15   11