Cumann Oidí Matamaitice na hÉireann **Irish Mathematics Teachers Association**

Foireann Mata 2015 **Team Maths 2015**

Babhta Réigiúnach **Regional Round**

BABHTA 1 ROUND 1

1) Write $2^n - 2^{n-1}$ in form 2^x

2) 120 people are having a meeting at which they are to be separated into equal-sized groups having at least three but no more than 12 to a group.
How many different group sizes are possible?

BABHTA 2 ROUND 2

1) Find the sum of the distances from one vertex of a square of side 2 cm to the midpoints of each of the sides of the square.

Answer in the form $a + b\sqrt{c}$, where a, b and $c \in N$.

2) A circle, which passes through the origin, cuts off intercepts of lengths 4 and 6 units on the positive x- and y- axes respectively. Find the equation of the circle.

Answer in form $x^2 + y^2 + 2gx + 2fy + c = 0$

BABHTA 3 ROUND 3

1) If a and b $\in$ R find the numerical value of a + b when

$$\frac{4\cos^2(\theta)-3}{1-2\sin(\theta)} = a + b\sin(\theta) \text{ , where } \text{Sin}(\theta) \neq \frac{1}{2}$$

2) For what positive value of k does the line x + y = k intersect the circle $x^2 + y^2 = 3$ at one point only?
Answer in the form $\sqrt{a}$, where a $\in$ N

BABHTA 4 ROUND 4

1) A rectangle with sides in the ratio 3 : 5 is inscribed in a circle. The four vertices of the rectangle are on the circle.
Calculate the ratio of the area of the rectangle to the area of the circle.
Answer in simplest form $\frac{a}{b\pi}$, where a and b $\in$ N

2) The following array of integers is called a ladder; each horizontal row is called a rung. Find the sum of the pair of integers on the 7th row.

1	1
2	3
5	7
12	17
.	.
.	.
.	.

BABHTA 5 ROUND 5

1) Find the sum of the roots of the equation

$$9^{2x+1} - 28.3^{2x} + 3 = 0$$

Answer in simplest form $\frac{a}{b}$, where a and b $\in$ Z

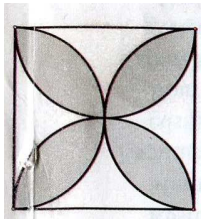
2) A box contains three coins; one coin is fair, one coin is two-headed, and one coin is weighted so that the probability of heads appearing is $\frac{1}{3}$. A coin is selected at random and tossed. Find the probability that a head will appear?

Answer in simplest form $\frac{a}{b}$, where a and b $\in$ N

BABHTA 6 ROUND 6

1) Two sides of a parallelogram have length 5 and the other two sides have length 7.
The length of one diagonal is 11.
Calculate the length of the other diagonal.
Answer correct to one decimal place.

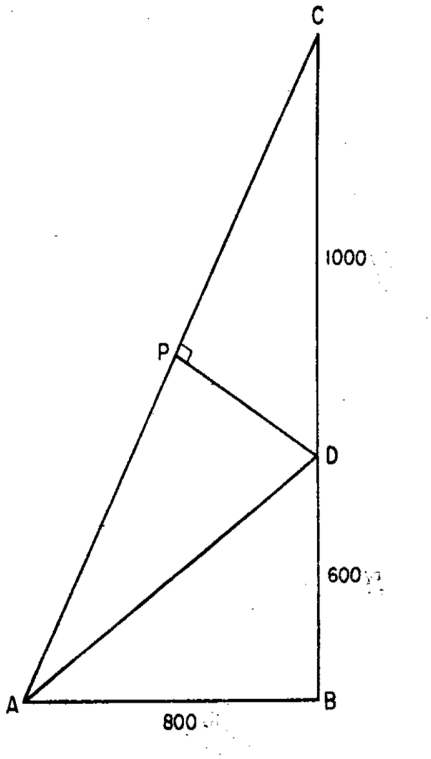
2) Four semicircles are drawn in the interior of a square using each side of the square as a diameter.
The area of the square is 64 square units.
Find the area of the shaded region.



Answer in form $a\pi$ - b, where a and b $\in$ N

BABHTA 7 ROUND 7

1) A and B are two points on a straight road and B is 800 m east of A.
D and C are two landmarks which are due north of B. It is known that $|BD| = 600$m and $|DC| = 1000$m.
Calculate the shortest distance, $|DP|$, from D to a straight road which joins AC.
Answer in simplest form $a\sqrt{b}$, where a and $b \in N$



2) If x is a positive real number and $(x+\frac{1}{x})^2 = 7$ find the value of $x^3+\frac{1}{x^3}$
Answer in simplest form $a\sqrt{b}$ where a and $b \in N$

3) The complex number z satisfies $z + |z| = 2 + 8i$.

Find the value of $|z|$.

4) Find the value of $x + y + z$ if $\frac{1}{xy}+\frac{1}{yz}+\frac{1}{zx}=12$ and $xyz=\frac{1}{18}$
Answer in simplest form $\frac{a}{b}$, where a and $b \in N$.

BABHTA 8 ROUND 8

1) Find all the solutions to the equation Cos(A) + Sin(A) = $\sqrt{\frac{3}{2}}$ in the domain O < A < π

Answers in terms of π.

2) Find the value of $x_2 + x_4 + x_6 + \ldots\ldots + x_{98}$ if $x_1, x_2, x_3 \ldots\ldots$ is an arithmetic progression with common difference 1, given that $x_1 + x_2 + x_3 \ldots\ldots + x_{98} = 137$.

3) The lengths of the sides of a triangle are 10, 17 and 21. What is the length of the shortest altitude of the triangle?

4) Find all the real positive values of p and r which satisfy the following equations:

$$p + pr + pr^2 = 26$$
$$p^2r + p^2r^2 + p^2r^3 = 156$$

Answers in the form (p , r).

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SCOILT TIEBREAK

1) The number 13 is prime. If you reverse the digits you also get a prime number, 31. Find the largest prime number that satisfies this condition if the sum of the two primes is 110.

2) If Alex stands on a table and Brian stands on the floor then Alex is 80 cm taller than Brian.
If Brian stands on the same table and Alex stands on the floor then Brian is 1 m taller than Alex.
How high is the table?

3) An equilateral triangle is cut into four equilateral triangles, each with a perimeter of 12 cm.
What is the perimeter, in cm, of the original equilateral triangle?

4) If the product $15^6 \times 28^5 \times 55^7$ was evaluated, it would end with a string of consecutive zeros.
How many zeros are in this string?

5) The numbers 1 to 10 are placed around a circle. Sue crosses out 1, then 4, and then 7. Continuing in a clockwise direction she crosses out every third number of those remaining, until only two numbers are left.
What is the sum of the two remaining numbers?

6) If $P = 3^x + 3^{-x}$ and $Q = 3^x - 3^{-x}$ what is the numerical value of $P^2 - Q^2$?

7) The graph of $5x - 3y - 7 = 0$ is translated 3 units up and 2 units to the right.
What is the equation of the new graph in the form
$ax + by + c = 0$, where a, b and $c \in Z$.

8) What is the 2015^{th} digit when $\frac{7}{13}$ is written as a decimal?

9) $a = \log_8(225)$ and $b = \log_2(15)$. Write a in terms of b.

10) a , b and c are real numbers which satisfy the following equations:

$$a - b + c = 2$$
$$b - c + a = -3$$
$$c - a + b = 5$$

Find the numerical value of $a + b + c$.

11) Find the altitude of the equilateral triangle whose area and perimeter have the same numerical value.

12) If $16^{x+1} = 3$, what is the value of 2^{4x+2}?

13) If $2^x = 3$, $3^y = 5$ and $5^z = 8$ what is the numerical value of the product xyz?

14) In the triangle ABC, $|AB| = 11$, $|AC| = 9$ and the length of the altitude from A to $[BC] = 7$. Calculate the length of the side $[BC]$.

Answer in simplest form $a\sqrt{b}$, where a and b $\in$ N.

Answers Team Maths Regional Round 2015

Round 1 **Q1** 2^{n-1} **Q2** 7

Round 2 **Q1** $2+2\sqrt{5}$ **Q2** $x^2+y^2-4x-6y=0$

Round 3 **Q1** **3** **Q2** $\sqrt{6}$

Round 4 **Q1** $\frac{30}{17\pi}$ **Q2** 408

Round 5 **Q1** $-\frac{1}{2}$ **Q2** $\frac{11}{18}$

Round 6 **Q1** 5.2 **Q2** $32\pi - 64$

Round 7 **Q1** $200\sqrt{5}$ **Q2** $4\sqrt{7}$
Q3 17 **Q4** $\frac{2}{3}$

Round 8 **Q1** $\frac{\pi}{12}, \frac{5\pi}{12}$ **Q2** 93
Q3 8 **Q4** $(18, \frac{1}{3})$ **and** $(2, 3)$

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Tiebreak

Q1 73 **Q2** 90 **Q3** 24 **Q4** 10 **Q5** 10

Q6 4 **Q7** $5x - 3y - 8 = 0$ **Q8** 6 **Q9** $a = \frac{2}{3}b$

Q10 4 **Q11** 6 **Q12** $\frac{3}{4}$ **Q13** 3 **Q14** $10\sqrt{2}$