Irish Maths Teachers Association

# Team Math Final – 2013

## University of Limerick

## Saturday April 13th, 2013

Foireann Mata 2013 (An Chraobh) :     Team Maths 2013 (Final)

# BABHTA 1    –    ROUND 1

**1)** When $(x + 3)(3x – t)(2x – 1)$ is multiplied out, the coefficient of $x^2$ is 5.

What is the value of $t$ ?

**2)** Find the value of $\dfrac{2^1 + 2^0 + 2^{-1}}{2^{-2} + 2^{-3} + 2^{-4}}$ in its simplest form.

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# BABHTA 2    –    ROUND 2

**1)**    What is the value of $\sqrt[3]{2\sqrt{2}}$ ?

**2)**    At an airport, there is a moving walkway 500metres long, which moves at a speed of 4km/h.
Alice and Bob step on the walkway at the same time. Alice walks at a speed of 6 km/h on the walkway while Bob stands still. When Alice comes to the end of the walkway, how far behind her is Bob?

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# BABHTA 3    –    ROUND 3

**1)**    Find all the roots of $8x^3 - 44x^2 + 86x - 65 = 0$, given that one root is $\frac{1}{2}(3 - 2i)$.

**2)**    A fair coin is tossed five times.
What is the  probability of obtaining at least three consecutive heads?
Answer in the form $\frac{a}{b}$, where $a$ and $b \in \mathrm{N}$.

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## BABHTA 4 – ROUND 4

**1)** Using the usual notation, find the radius length of the circumcircle of the triangle ABC, where $a = 5$ cm, $b = 6$ cm and $c = 8$ cm.
Answer to nearest cm.

**2)** If $xy = 2$, $yz = 3$ and $zx = 5$, what is the value of $x^2 + y^2 + z^2$ ?
Answer in the form $\dfrac{a}{b}$, where $a$ and $b \in$ N.

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# BABHTA 5   –   ROUND 5

**1)**   If *a, b* and *c* are positive integers such that

$$a + \cfrac{1}{b + \cfrac{1}{c}} = \frac{37}{16}, \text{ calculate the value of } a + b + c.$$

**2)**   Evaluate, correct to 3 significant figures,

$$\int_1^2 \frac{\left(e^{2x} - e^{-2x}\right)dx}{2}$$

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# BABHTA 6   –   ROUND 6

**1)**   If $2000^2 - 1996^2 = 111ak^2$, where $a$ and $k$ are integers, what is the maximum of $k - a$?

**2)**   The first digit of a six-digit number is 1. This digit, 1, is now moved from the first digit position to the end so that it becomes the last digit. The new six-digit number is now 3 times larger than the original number. What was the original number?

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# BABHTA 7   –   ROUND 7

**1)** If $x$ and $y$ are real numbers such that $0 < x < y$ and $x^2 + y^2 = 6xy$, find the value of $\dfrac{x + y}{x - y}$

**2)** Arrange in ascending order of magnitude:

$$2^{800}, \quad 3^{600}, \quad 5^{400}, \quad 6^{300}.$$

**3)** A solid circular cylinder is to be made from 10 cm³ of material. Calculate, correct to 2 decimal places, the height of the cylinder when the surface area is a minimum. (Use the most accurate value of $\pi$).

**4)** When the digits of a two-digit number are reversed, the result is another two-digit number that is 62·5% smaller than the original number. What is the original number?

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## BABHTA 8   –   ROUND 8

**1)**  Find the value of *x*, in the form $\dfrac{a}{b}$ where *a* and $b \in N$,

when  $8^{3x+1} - 8^{3x} = 448$

**2)**  When  $A + B + C = 180^\circ$,
$Sin(2A) - Sin(2B) + Sin(2C) = k Cos(A)Sin(B)Cos(C)$.
Find the value of *k*.

**3)**  Find all the positive solutions to the equation
$(\log_{10}(x^2))^2 = \log_{10}(x^4)$

**4)**  A hotel cleaner has 8 master keys at home to open all rooms in the hotel. Each room can be opened by just one of these keys.
If 40% of the rooms are left unlocked, what is the probability that the cleaner can get into a specific room if she selects three keys at random before leaving home to go to work?

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# SCOILT   –   TIEBREAK

**1)** What is the value of $\dfrac{-2x^4}{(-2x)^4}$ ?

**2)** The lengths of the altitudes of a parallelogram are 3 cm and 5 cm and its perimeter is 32 cm. What is the area of the parallelogram?

**3)** The ratio of the volumes of two cubes is 1 : 8.  What is the ratio of their surface areas?

**4)** Find the value of $\dfrac{\dfrac{1}{5}+\dfrac{1}{7}}{\dfrac{2}{5}+\dfrac{2}{7}}$

**5)** The point $(a, -2)$ is on the line joining the points $(0, 6)$ and $(-3, 0)$.
Find the value of $a$.

**6)** What is the numerical value when the sum of the first 80 odd natural numbers is subtracted from the sum of the first 80 even natural numbers? An even natural number begins with 2.

**7)** The odd numbers are arranged in a triangle as shown:

```
                    1
               3         5
          7         9         11
      13        15        17        19
```
…………………………………………………………..
If the pattern continues in this form what is the sum of the numbers in the 12$^{th}$ row?

**Turn over for 7 more questions ➜**

**8)**   What is the numerical value of $\text{Sin}(70^0)\text{Cos}(20^0)+\text{Cos}(70^0)\text{Sin}(20^0)$ ?

**9)**   A fair die is tossed 7 times. What is the probability that a 5 or a 6 occurs exactly 3 times?  Answer correct to 2 decimal places.

**10)**   Let X be a random variable with standard normal distribution.
Find $p(\text{-}1\cdot37 < \text{X} < 2\cdot01)$

**11)**    Find the equations of the tangents to the circle $x^2 + y^2 - 2x + 8y - 23 = 0$ which have a slope of 3. Answer in the form $ax + by + c = 0$, where $a,\ b$ and $c\ \in \text{N}$.

**12)**   In a list of five numbers, the first number is 2 and the last number is 12. The product of the first three numbers is 30, the product of the three numbers in the middle is 90 and the product of the last three numbers is 360.
What is the value of the middle number?

**13)**   What is the last non-zero digit of the product $2^{59} \times 3^4 \times 5^{53}$ ?

**14)**   The sum of the digits of a nine-digit integer is 8. What is the product of these digits?

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# Answers – Final 2013

| Round 1 | Round 2 |
|---|---|
| **(1)** 5 | **(1)** $\sqrt{2}$ |
| **(2)** 8 | **(2)** 300 metres |

| Round 3 | Round 4 |
|---|---|
| **(1)** $\frac{1}{2}(3 \pm 2i)$, 2·5 (or equivalent) | **(1)** 4 |
| **(2)** $\frac{1}{4}$ | **(2)** $\frac{361}{30}$ |

| Round 5 | Round 6 |
|---|---|
| **(1)** 10 | **(1)** 11 |
| **(2)** 11·8 | **(2)** 142857 |

| Round 7 | Round 8 |
|---|---|
| **(1)** $-\sqrt{2}$ | **(1)** $\frac{2}{3}$ |
| **(2)** $6^{300}$, $2^{800}$, $5^{400}$, $3^{600}$ | **(2)** 4 |
| **(3)** 2·34 cm. | **(3)** 1 and 10 |
| **(4)** 72 | **(4)** $\frac{5}{8}$ or equivalent |

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## Answers – Tiebreak, Final 2013

1)    $-\dfrac{1}{8}$

2)    30

3)    1 : 4

4)    $\dfrac{1}{2}$ or equivalent

5)    $-4$

6)    80

7)    $1728 = 12^3$

8)    1

9)    0·26

10)    0·8925

11)    $3x - y + 13 = 0$ and $3x - y - 27 = 0$

12)    5

13)    4

14)    0