OLLSCOIL NA hÉIREANN, CORCAIGH
THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH
UNIVERSITY COLLEGE, CORK

TEAM MATH FINAL 2010

ROUND 1

Time allowed: **Six** minutes

Q.1 Express $x = \dfrac{2t+1}{3t-1}$, $y = \dfrac{t+2}{3t-1}$ in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{R}$.

Q.2 Evaluate $\lim\limits_{x \to 0} \dfrac{\sin 5x - \sin x}{3x}$

ROUND 2

Time allowed: **Six** minutes

Q.1 Find the value of x that satisfies the equation

$$\log_3 x + \log_9 16 = 0, \text{ where } x \in \mathbb{Q}.$$

Q.2 The line $x - y + 3 = 0$ passes through the centre of a circle.
The points $A(-1, 2)$ and $B(2, -4)$ are on the circle.
Write the equation of the circle in the form $x^2 + y^2 + 2gx + 2fy + c = 0$, where $g, f, c \in \mathbb{R}$.

ROUND 3

Time allowed: **Six** minutes

Q.1 Solve the equation $\sqrt{x} + \dfrac{24}{\sqrt{x}} = 10$, where $x > 0$.

Q.2 Write $\cos^4 x - \sin^4 x$ in the form $a \cos bx$, where $a, b \in \mathbb{Z}$.

ROUND 4

Time allowed: **Six** minutes

Q.1 Find the values of θ that satisfy the equation

$$|1 + \cos 2\theta + i \sin 2\theta| = \sin 2\theta,$$

where $0° \leqslant \theta \leqslant 360°$.

Q.2 Find the equations of the two lines that pass through the point $(2, -1)$ and make an angle of $60°$ with the x-axis.

Write your answers in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{R}$, using surd values where relevant.

ROUND 5

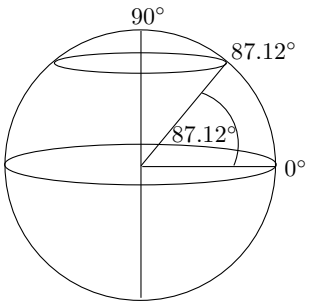
Time allowed: **Six** minutes

Q.1 A committee of *five* is to be formed from a group of *nine* people which includes Jack and Jill. Jack will **not** serve on the committee without Jill, but Jill will serve without Jack.

In how many ways can the committee be formed?

Q.2 Taking the earth as a sphere, the diagram shows the circular parallels of latitude $0°$ and $87.12°$.

Taking the length of latitude $0°$ to be 40 000 km, calculate the length of latitude $87.12°$, correct to the nearest km.



ROUND 6

Time allowed: **Six** minutes

Q.1 Writing x^3 as $x^2 \cdot x$ or otherwise, evaluate

$$\int_{\sqrt{2}}^{\sqrt{7}} \frac{x^3}{\sqrt{2 + x^2}}\, dx.$$

Write your answer in the form $\dfrac{a}{b}$, where $a, b \in \mathbb{Z}$.

Q.2 An ATM machine contains thirty notes.
There is at least one €10, one €50 and one €100 note in the machine.
These are the only notes in the machine and the total value of the thirty notes is €2010.

How many of each of the notes are in the machine?

Write, clearly, the two sets of possible answers.

ROUND 7

Time allowed: **Six** minutes

Q.1 Find, correct to two decimal places, the value of x that satisfies

$$2^{2x} - 2^{x+3} - 20 = 0, \text{ where } x \in \mathbb{R}.$$

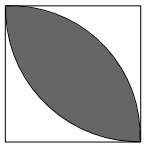
Q.2 The volume of a spherical balloon is increasing at the rate of $20\,\text{cm}^3$ per second, i.e. $\dfrac{dv}{dt} = 20$.

Find $\dfrac{dr}{dt}$ when $r = 1\,\text{cm}$.

Write your answer in the form $\dfrac{a}{\pi}$, where $a \in \mathbb{Z}$.

Q.3 The diagram shows a square of side $4\,\text{cm}$ in length and two arcs, each of radius $4\,\text{cm}$ and with centres at opposite vertices of the square.

Calculate, in terms of π, the area of the shaded region between the two arcs.



Q.4 Simplify $\begin{pmatrix} \cos 60^\circ & -\sin 60^\circ \\ \sin 60^\circ & \cos 60^\circ \end{pmatrix}^{11}$.

Write your answer in the form $\dfrac{1}{2}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where $a, b, c, d \in \mathbb{R}$, using surd values where relevant.

ROUND 8

Time allowed: **Six** minutes

Q.1 The result, either ***win***, ***lose*** or ***draw***, of each of four matches is forecast.

Each of the possible results is equally likely for each match.

Find the probability that the forecast is correct for exactly two of the four matches.

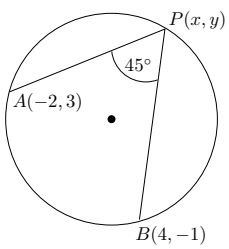
Q.2 Evaluate $\displaystyle\int_3^8 \frac{1}{\sqrt{16 + 6x - x^2}}\,dx$.

Write your answer in terms of π.

Q.3 $A(-2, 3)$, $B(4, -1)$ and $P(x, y)$ are points on a circle such that $|\angle APB| = 45^\circ$.

Using the formula $\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2}$, or otherwise, find the equations of two circles that satisfy these conditions.

Write your answers in the form $x^2 + y^2 + 2gx + 2fy + c = 0$, where $g, f, c \in \mathbb{Z}$.



Q.4 α and β are the roots of the equation $x^2 - 3x + 1 = 0$.

Calculate the value of $\alpha - \beta$, where $\alpha > \beta$.

Write your answer in the form $\sqrt{a}$, where $a \in \mathbb{Z}$.

TIE BREAK 1

Maximum time allowed: **Six** minutes

The team may benefit if the answers are handed up before those of other teams.

Q.1 Evaluate

$$\int_1^4 \cos^2 x\, dx + \int_1^4 \sin^2 x\, dx.$$

Q.2 $z = 2 - 3i$ is a root of

$$z^2 + az + b = 0, \text{ where } a, b \in \mathbb{R}.$$

Find the value of a and the value of b.

Q.3 l is the line $2x + 3y - 1 = 0$ and k is the line $x - 2y - 5 = 0$.

Find the equation of the line that passes through the origin and through the point $l \cap k$.

Write your answer in the form $ax + by = 0$, where $a, b \in \mathbb{Z}$.

Q.4 Suppose $y = \log_{e^2} x$. Find $\dfrac{dy}{dx}$.

TIE BREAK 2

Maximum time allowed: **Six** minutes

The team may benefit if the answers are handed up before those of other teams.

Write

$$\left(\sin\frac{\pi}{3} + i\cos\frac{\pi}{3}\right)^{20}$$

in the form $\dfrac{x}{2} + i\dfrac{y}{2}$, where $x, y \in \mathbb{R}$, using surd values where relevant.

FINAL 2010: ANSWERS

Round	Question 1	Question 2	Question 3	Question 4
1	$7x - 5y - 3 = 0$ or equivalent	$\frac{4}{3}$		
2	$x = \frac{1}{4}$	$x^2 + y^2 + 17x + 11y - 10 = 0$		
3	$x = 16$ and $x = 36$	$1 \cos 2x$		
4	$\theta = 90°$ and $\theta = 270°$	$\sqrt{3}x - 1y - 1 - 2\sqrt{3} = 0$ and $\sqrt{3}x + 1y + 1 - 2\sqrt{3} = 0$ or equivalent		
5	91	2010 km		
6	$\frac{13}{3}$	$1 \times$ €10, $18 \times$ €50, $11 \times$ €100 and $6 \times$ €10, $9 \times$ €50, $15 \times$ €100		
7	$x = 3.32$	$\frac{5}{\pi}$ cm/sec	$8\pi - 16\,\text{cm}^2$	$\frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}$
8	$\frac{8}{27}$	$\frac{\pi}{2}$	$x^2 + y^2 + 2x + 4y - 21 = 0$ and $x^2 + y^2 - 6x - 8y - 1 = 0$	$\alpha - \beta = \sqrt{5}$
TIE 1	3	$a = -4$, $b = 13$	$9x + 17y = 0$ or equivalent	$\frac{dy}{dx} = \frac{1}{2x}$
TIE 2	$-\frac{1}{2} - i\frac{\sqrt{3}}{2}$			